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Chuanjiang Luo, Jian Sun and Yusu Wang

Integral Estimation from Point Cloud in $\Re^d$: a Geometric View

Integration over a domain, such as a Euclidean space or a Riemannian manifold, is a fundamental problem across scientific fields. Many times, the underlying domain is only accessible through a discrete approximation, such as a set of points sampled from it, and it is crucial to be able to estimate integral in such discrete settings. In this paper, we study the problem of estimating the integral of a function over a k-submanifold embedding in $\Re^d$, from its function values over a set of sample points. Previously, such estimation is usually obtained in a statistical setting, where input data is usually assumed to be drawn from certain probabilistic distribution. Our paper considers arbitrary point clouds data (PCD), and approaches the problem from a geometric point of view. Specifically, we model the integral as a weighted sum, and propose two weighting schemes: the Voronoi and the Principle eigenvector weighting schemes. The running time of both methods depends mostly on the intrinsic dimension of the underlying manifold, instead of on the ambient dimensions. We show that the estimation based on the Voronoi scheme converges to the true integral (explicit error bound is given) under the standard (ε, δ)-sampling condition. This is the first result of this sort for estimating integral from general PCDs. For the Principle eigenvector scheme, although no theoretical guarantee is established for it, we show its connection to the heat diffusion operator and illustrate several justifications behind its construction. Experiments show that both new methods consistently out-perform common statistical methods under various sampling conditions.